1. Introduction
The reflective liquid crystal display (RLCD) modes with a single polarizer have been discussed in many literatures. RLCD can be generally categorized into two types, quarter-wave mode and half-wave mode. In quarter-wave mode, the LC cell acts as a quarter wave plate at voltage-off state and as a homeotropic layer at voltage-on state. TN-ECB mode [1~3], SCTN mode [4] and some MTN [5] modes working in zero reflectance condition at voltage-off state can be classified into this type. In half-wave mode, the LC cell acts as a half wave plate at voltage-off state and quarter wave plate at voltage-on state. The so called RTN mode [6] can be considered this type.

In general, the two modes have their merits and demerits. It can be easily understood that, it is dispersive at voltage-off state for quarter-wave mode, and at both voltage-on and voltage-off states for half-wave mode. Ideally, there should be no dispersion at voltage-on state of quarter-wave mode due to homeotropic alignment. Thus, the quarter-wave mode should possess better contrast ratio than that of half-wave one in direct-view RLCD. On the other hand, at voltage-on state of half-wave mode, LC molecules do not need to reach a homeotropic state, so the driving voltage is relatively low compared to that of quarter-wave mode.

To achieve best contrast ratio, Chen et al. used a parameter space method [3], i.e. by fixing the polarizer angle and observing the two-dimensional reflectance contour in retardation versus LC twist plot for a single wavelength, followed by examining the contrast ratio by overlapping the contrast ratio on the two-dimensional reflectance contour [7]. This method is elegant in determining the RLCD operating parameters (polarizer angle \( \phi \), twist angle \( \alpha \), and retardation \( d\Delta n \)) with one parameter fixed. However, the search is still not thorough in freeing all parameters. In this paper, we will introduce a method of optimizing the contrast ratio by examining all possible combinations of \((\alpha, \phi, d\Delta n)\) that satisfy zero reflectance. Furthermore, this possible combinations of \((\alpha, \phi, d\Delta n)\) can be plotted in a two-dimensional plot explicitly. The dispersion can also be considered by plotting the reflectance for zero reflectance combination of \((\alpha, \phi, d\Delta n)\), from which, the ultimate optimized parameters of \((\alpha, \phi, d\Delta n)\) can be obtained. The results can be applied in direct-view RLCD and single panel projection display, where dispersion has to be minimized. We shall use quarter-wave mode as an example, however, the method can be extended to half-wave mode as well.

![Figure 1](image-url)
points of $\alpha = 0$, $(\phi, \Delta nd) = (63.6^\circ, 0.194 \mu m)$, $(190.8^\circ, 0.583 \mu m)$, corresponds to two TN-ECB modes [1-3]. It can also be verified that the low twist MTN modes also fit in the graph nicely [5]. It is interesting that there is an intersection point of the $\alpha$ curve and $\phi$ curve, which means the polarizer angle

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corresponds to two TN-ECB modes [1~3]. It can also be verified that the low twist MTN modes also fit in the graph nicely [5]. It is interesting that there is an intersection point of the $\alpha$ curve and $\phi$ curve, which means the polarizer angle

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is exactly the same as twist angle at this position. The curve has a sudden change at $\Delta nd = 0.447 \mu m$, which corresponds to the alternation of two reflectance minima at $\phi = 0$ and $\phi = 210^\circ$.

Figure 3(a) and (b) show the reflectance versus twist angle $\phi$ in the condition of $\Delta nd < 0.447 \mu m$ ( $\Delta nd = 0.4439 \mu m$) and $\Delta nd > 0.447 \mu m$ ( $\Delta nd = 0.4508 \mu m$) respectively. The corresponding $\alpha$ is also shown in the figure. It can be seen that there are two troughs of the reflectance curve, when $\Delta nd$ crosses 0.447 $\mu m$, the lower trough, which is the reflectance minimum, change from $\phi = 0$ to $\phi = 210^\circ$, which causes the jump in Figure 2.

There are two turning points at $\Delta nd = 0.41 \mu m$ and 0.57 $\mu m$, between which, there is no solution for zero reflectance. The minimum reflectance versus $\Delta nd$ is shown in figure 4. It can be clearly seen that, the minimum reflectance in the range of 0.41 $\mu m < \Delta nd < 0.57 \mu m$ does not equal to zero. Only in the range of 0.1375 $\mu m < \Delta nd < 0.41 \mu m$ and $\Delta nd > 0.57 \mu m$, it is zero reflectance.

To obtain best contrast RLCD in direct view application, dispersion has to be taken into consideration. Using the optimized parameters of $(\alpha, \phi, d\Delta n)$ shown in figure 2, we can further calculate the reflectance for blue light ($\lambda = 450$ nm) and red light ($\lambda = 650$ nm), which are shown in figure 5(a) and (b) respectively. Figure 5(c) shows the average reflectance of red and blue light versus $\Delta nd$. It can be seen that, in the range of 0.14 $\mu m < \Delta nd < 0.25 \mu m$, the dispersion is relatively small and even. For larger value of $\Delta nd$, the dispersion is quite serious. The sudden changes in the curves of figure 5(a), (b) and (c) correspond to the jump in figure 2. Reflectance can be calculated for the whole visible spectrum for a better evaluation. It is worth pointing out that, the optimized range of $\Delta nd$ does not contradict to any existing knowledge about RLCD.

Figure 2. $\alpha$ and $\phi$ versus $\Delta nd$ plot with minimized reflectance for 550 nm light.

Figure 4. The minimum reflectance versus $\Delta nd$ for 550 nm light using the parameters shown in Figure 2.

Figure 3. The alternating minimum peaks of reflectance when $\Delta nd$ crosses 0.447 $\mu m$ in Figure 2.

Figure 5. (a) Reflectance for blue light ($\lambda = 450$ nm) with $\Delta nd = 0.4439 \mu m$ and $\alpha = 45^\circ$. (b) Reflectance for red light ($\lambda = 650$ nm) with $\Delta nd = 0.4508 \mu m$ and $\alpha = 7.6^\circ$. (c) Average reflectance for red and blue light versus $\Delta nd$. The sudden changes in the curves of figure 5(a), (b) and (c) correspond to the jump in figure 2.
In conclusion, we have shown a method utilizing numerical calculation to study the reflective modes in searching all possible combinations of \((\alpha, \phi, d\Delta n)\). Our method provides a comprehensive and thorough way to refine reflective LCD mode. Moreover, it can be shown clearly that the optimized reflective modes exist in a range of \(nd\Delta\) (between 0.14 and 0.25 \(\mu m\) for quarter-wave mode). Because \(nd\Delta\) is determined by liquid crystal and spacer dimension, \(\phi\) and \(\alpha\) can be set during manufacturing process of LC cell assembly. So a wider range of liquid crystal and spacer can be chosen in practice (with different \(\alpha\) and \(\phi\)). It is worth pointing out that similar method can also be employed to analysis half-wave mode in both the voltage-on and voltage-off states.

4. Acknowledgements
This work is supported by Public Sector R&D Funding (#0121010027) from Agency for Science, Technology and Research (A*STAR), Singapore.

5. References

Figure 5. Reflectance versus \(\Delta nd\) for red and blue light in the condition of optimized parameters for green light \((\lambda = 0.55 \mu m)\); (a) Curve of reflectance versus \(\Delta nd\) for red light \((\lambda = 0.65 \mu m)\); (b) Curve of reflectance versus \(\Delta nd\) for blue light \((\lambda = 0.45 \mu m)\); (c) The average reflectance of red and blue light versus \(\Delta nd\).