

Dynamic switching of optical vortices with dynamic gamma-correction liquid crystal spiral phase plate

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Abstract: Generation and dynamic switching of optical vortices from charge 1 up to charge 6 at wavelength of $0.6328\mu\text{m}$ by one dynamic gamma-correction liquid crystal spiral phase plate are reported. The liquid crystal spiral phase plate comprises 46 slices, which are driven by a 16-channel voltage output card. The spiral phase plate was designed based on the relationship between the topological charge purity of an optical vortex generated by the spiral phase plate and the total number of slices. The calculation results show that a minimum slice number of 44 is required for generating optical vortices up to charge 6 with charge purity above 94%.

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1. Introduction

Optical vortices have been widely used in the study of optical tweezers [1-5], trapping and guiding of cold atoms [6-8], rotational frequency shift [9, 10] and entanglement states of photons [11] etc.

There are several approaches to generate an optical vortex. The most common approach is the computer generated hologram [12, 13], in which an optical vortex can be obtained as a diffraction beam. The second approach is that, through an asymmetric structure inside a laser cavity, a high order Hermit Gaussian beam is generated, which is then converted into an optical vortex by two astigmatic lenses [14]. There are also papers on changing the structure inside laser cavity to obtain optical vortices without using cylindrical lens mode converter [15]. The third one is using a spiral phase plate (SPP) to convert a Gaussian beam into an optical vortex.

The spiral phase plate approach has the merit of high conversion efficiency comparing with the hologram approach, and it can be easily handled comparing with the cylindrical lens

mode converter. SPPs are not only used for generating optical vortices, it has also found applications in generating radially polarized beams [16] and edge contrast enhancement in microscopy [17]. Various kinds of SPPs published hitherto are listed as follows, (1) a large pitch spiral phase plate dip into a refractive index matched solution [18]; (2) fabricating multi-steps on the silica material though multi-etching process [19]; (3) by use of excimer laser ablation to make multi-steps on polyimide substrate [20]; (4) employing a diamond tool on a high-precision computer driven lathe to make a mold at first, into the mold a polymer is cast, which is then UV cured [21]; (5) using a deformed cracked plexiglass plate [22]; (6) electron-beam writing in photo resist [23]; (7) multi-stage vapor deposition [24]; (8) using liquid crystal (LC) cell as a phase modulator [25, 26]. In this paper, we shall present a specially designed LC SPP. Using only one of such SPP, optical vortices (at 0.6328 μm wavelength) of topological charge from 1 up to 6 with high quality are successfully generated and can also be dynamically switched by a multi-channel voltage output card. Comparing with other kinds of SPPs, the LC SPP presented here has advantages of high quality, low cost and easy to fabricate, being able to generate optical vortices with various charges (even if it is not an integer but a fraction) by only one SPP, adaptable to any visible wavelength, program controllable.

2. Relationship between beam purity and slice number

Multi-level SPP, comprising limited number of phase slices, is using discrete phase steps to imitate a continuous phase ramp. Apparently, it is an approximation. Using Fourier method, we shall show below the relationship between beam purity and the total slice number.

Denoting s as the total slice number of SPP, ϕ as the azimuthal angle, then the effect of SPP with charge m can be represented by:

$$f(\phi) = e^{i \cdot \text{mod}(\frac{s\phi}{2\pi}) \cdot \frac{2m\pi}{s}} \quad (1)$$

where $\text{mod}(\frac{s\phi}{2\pi})$ refers to the maximum integer less than $\frac{s\phi}{2\pi}$, and $0 \leq \phi \leq 2\pi$.

According to the Fourier transformation theory, $f(\phi)$ can be written as

$$f(\phi) = \sum_{n=-\infty}^{+\infty} c_n \exp(jn\phi), \text{ where } n \text{ is an integer, } -\infty < n < +\infty, \text{ and}$$

$$c_n = \frac{1}{2\pi} \int_0^{2\pi} f(\phi) \exp(-jn\phi) d\phi = \frac{1}{2\pi} \int_0^{2\pi} e^{i \cdot \text{mod}(\frac{s\phi}{2\pi}) \cdot \frac{2m\pi}{s}} \exp(-jn\phi) d\phi \quad (2)$$

Assuming a laser beam with an electric field of $F(\rho, z)$ incidents on the SPP, the electric field of the emerged beam immediately after the SPP can be written as

$$u(\rho, \phi, z) = F(\rho, z) \sum_{n=-\infty}^{+\infty} c_n \exp(jn\phi) \quad (3)$$

This is a combination of infinite Fourier components, each component represents an optical vortex with certain charge, c_n is the amplitude of the charge n component. For a SPP with charge m , only the component $c_m \exp(jm\phi)$ is desired, the others are unwanted. The percentage of all those unwanted components, R , can be written as

$$R = 1 - |c_m|^2 \quad (4)$$

Assuming $R < 6\%$ (Such a ratio should be good enough for most application, such as optical tweezers and cold atom guiding), from Eqs. 1 and 4, we can calculate the minimum requirement of the total slice number for a certain charge. The calculation results are tabulated in Table 1.

Table 1. Minimum slice number of a SPP and the corresponding topological charge of a vortex beam generated with over 94% purity.

SPP Charge number	1	2	3	4	5	6	7	8
Minimum Slice number	8	15	22	30	37	44	52	59

It is worth measuring the beam purity experimentally. By measuring the beam purity, the orbital angular momentum spectrum of a light beam can be obtained [27~29]. However, the spectrometer must have high resolution to verify the high beam purity (94%) in our case. This work is in progress.

3. Liquid crystal spiral phase plate module

Using a cell gap of $18 \mu\text{m}$, and LC with a birefringence of 0.23, we fabricated a LC SPP that can generate topological charge up to 6 for a $0.6328 \mu\text{m}$ He-Ne laser light beam. A total slice number of 46 was used, which satisfied the requirement of minimum slice number for generating a 94% pure beam with charge 6 in Table 1.

Figure 1 shows the relationship between phase retardation of the liquid crystal cell and the applied voltage. The measurement was done using a Michelson interferometer. By using polynomial curve fitting, the voltages applied to all slices for generating a vortex beam of a certain charge can be determined from Fig. 1.

The driving was realized by using a multi-channel analog voltage output card. To drive 46 slices, normally a 46-channel voltage output card is needed. However, it is possible to use a voltage output card with less channel number to achieve the same performance. Actually, we used only one 16-channel voltage card to drive it. If the whole dynamic region in Fig. 1 is evenly divided into 15 regions, then in each region, the retardation can be regarded as linearly dependent on driving voltage, and linearly distributed voltage across slices can be realized by connecting each pair of adjacent slices with a resistor, a series of resistors with the same resistance acts as a linear resistors voltage divider. These resistors can be fabricated by patterning ITO layer. By doing so, the 15 retardation regions with 46 slices can be driven by only one multi-channel output card with 16 channels.

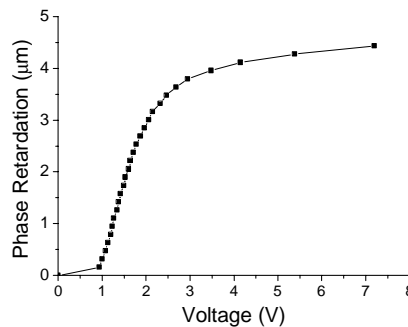


Fig. 1. Phase retardation versus driving voltage of the LC cell.

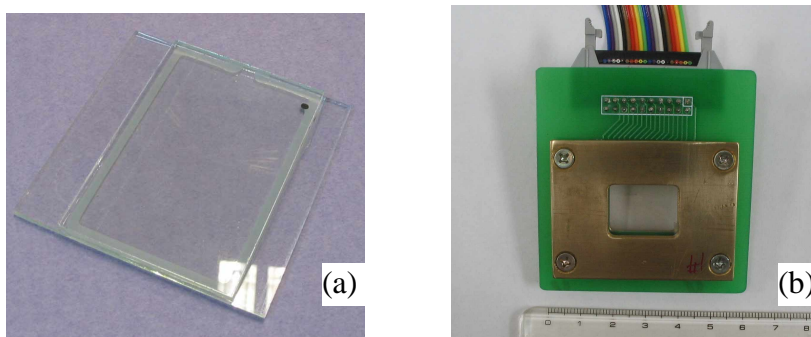


Fig. 2. (a). Photograph of a LC SPP cell; (b). Photograph of an assembled LC SPP module.

It is worth mentioning that, in our experiment, especially thick cell gap up to $18\ \mu\text{m}$, and high birefringence ($\Delta n = 0.23$) liquid crystal material were used. Besides, anti-parallel alignment of liquid crystal molecules was employed.

The fabrication process of LC SPP cell is similar to the assembly process of a liquid crystal display cell. The fabricated LC SPP cell was assembled into a module for easy connection to multi-channel output card. The LC SPP cell and LC SPP module fabricated are shown in Figs. 2(a) and 2(b) respectively. The LC SPP proved to be robust, reliable and convenient in use. It is worth mentioning that we have developed a fabrication process in our lab to produce LC SPPs in batches with consistent quality.

4. Results and discussion

The direct method to verify an optical vortex is to interfere it with a plane wave [30]. The experiment setup is actually a Mach-Zehnder interferometer, which is shown in Fig. 3.

By applying different voltage, optical vortex with various charges from 1 up to 6 can be obtained. Figure 4 shows the fork like interference patterns corresponding to optical vortices with different topological charges. A fork like interference pattern is formed when the two interference beams are at a small angle with each other. If the two beams are coaxial, the interference pattern will turn into a radial spokes pattern, which is shown in Fig. 5. We also observed the Fraunhofer diffraction pattern of Gaussian beam passing through LC SPP, the experiment setup is depicted in Fig. 6; the diffraction patterns are shown in Fig. 7. All these pictures were taken at the same focus length.

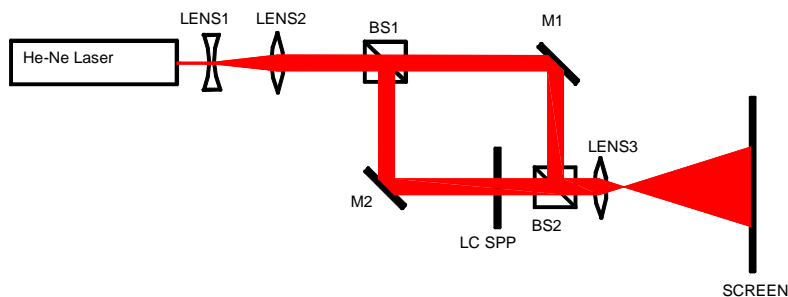


Fig. 3. The setup for verification of optical vortex.

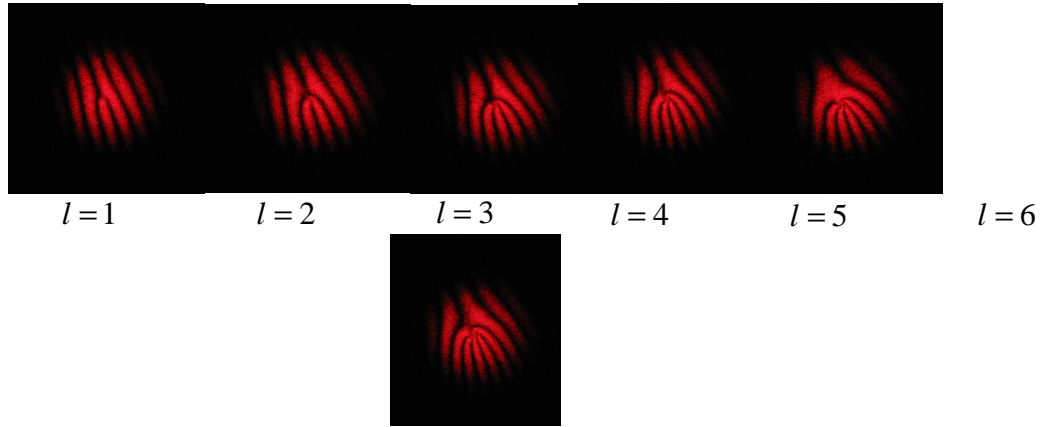


Fig. 4. Fork like interference patterns corresponding to charges from 1 up to 6.

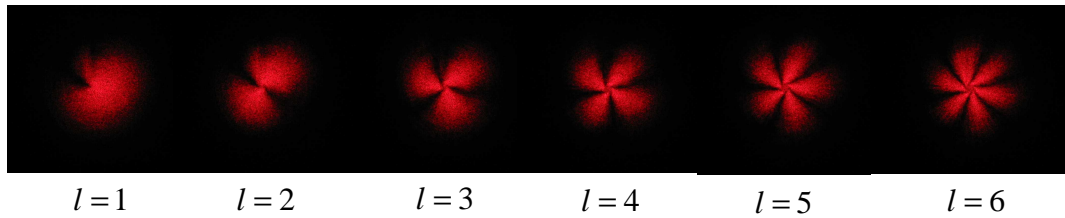


Fig. 5. (2.06MB) Movie of dynamic switching of radial spokes interference patterns from charges 1 to 6.

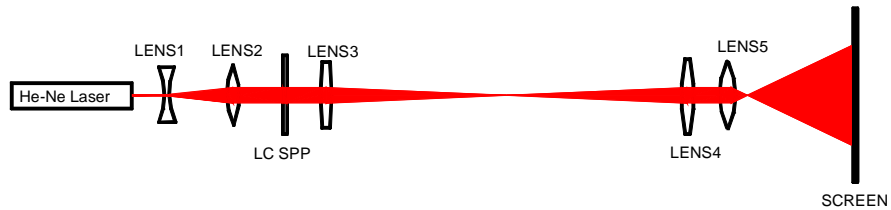


Fig. 6. Setup for observing Fraunhofer diffraction pattern of Gaussian beam passing through the LC SPP.

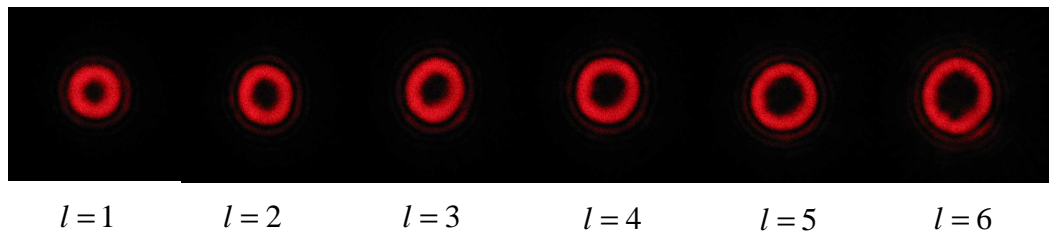


Fig. 7. (1.58MB) Movie of Fraunhofer diffraction patterns of Gaussian beams passing through LC SPP with charges from 1 to 6.

By properly coding the driving program, the charge of LC SPP can be dynamically switched. However, due to the thick cell gap and high viscosity LC used, change from one charge to another takes about one second. The dynamic switching of Laguerre-Gaussian beam of different charge is shown in the movies linked to Figs. 5 and 7, which show that the optical vortices are generated by the dynamic gamma-correction LC SPP with high quality.

Actually, in the center of the LC SPP, there is a small circle about 100 μm diameter in size, whose retardation cannot be controlled by applied voltage. However, this part of light beam is only a very small percentage of the total beam energy.

Generally, when a Gaussian beam passes through a spiral phase plate, the emerged beam is not a pure mode of Laguerre-Gaussian (LG) beam even though the topological charge of the SPP is an integer [31]. Actually, if the topological charge of SPP is l , then the beam emerged is composed of LG modes with the same topological charge l but different radial index, and the largest weight mode is LG_{0l} . For the modes with non-zero index, the diffraction pattern presents more than one ring, which can be seen in Fig. 7, however, the figure shows that most of the energy is within the first ring. So the impurity will not be a problem in most applications, such as optical tweezers, frequency shifter and cold atom guiding.

The dynamic gamma-correction LC SPP described here may find applications in optical tweezers. There are several kinds of beams, such as Gaussian beam and various order LG beam, have been employed in optical tweezers. With the LC SPP, it would not be difficult to combine all these beams together in one apparatus, and each beam may be dynamically switched in seconds. Moreover, the energy of the laser beam can be used in high efficiency compared to the commonly used hologram approach. All these merits would bring considerable convenience to optical tweezers related research.

The LC SPP can also be used to generate non-integer topological charge optical vortex, i.e., the l in the expression of $\exp(il\phi)$ is no longer an integer but a fraction, the term of $\exp(il\phi)$ represents the azimuth angle dependent wave front of an optical vortex. The photon of non-integer topological charge optical vortex possesses high dimensional entanglement, which may find applications in future optic communication [11]. The LC SPP described here are especially suitable for generation of optical vortices with non-integer topological charge of any value and dynamically switching between them.

As the fabrication process is based on lithography and LCD cell assembly, it is possible to fabricate large slice number and large size SPP at low cost. According to aforementioned analysis, a SPP with large slice number can improve the beam purity of a certain topological charge. By expanding light beam, large size LC SPP is possible to work in the condition of high intensity laser pulses.

It is worth mentioning that the LC SPP only works for linearly polarized light.

5. Conclusion

We designed and fabricated a dynamic gamma-correction LC SPP. Using this kind of SPP, high quality optical vortices with topological charge from 1 to 6 can be generated by only one SPP. the charge of the optical vortex can be dynamically switched in about 1 second. The LC SPP is able to adapt to any visible wavelength. The fabrication process is quite similar to that of a normal LCD cell, so it is low cost and reliable. These advantages are distinct compared to other types of SPP.